Multigranulation with Different Grades Rough Set in Ordered Information System

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***Abstract*—The the graded rough set and multi-granulation rough set are two significant extended rough set models, both constructed on the the indiscernibility relations. The purpose of this paper is to study the good points of graded rough set in the multi-granulation environment which in different granule have different grades in ordered information system. Three new types multi-granulation with different grades rough set models are proposed, which include optimistic multi-granulation with different grades, pessimistic multi-granulation with different grades and the mean multi-granulation with different grades rough set. Then, their principal structure are studied, and their basic properties are obtained as well. Finally, we study a case about students' achievement estimate the performance of the proposed properties. In the viewpoint of granular computing, our study extension the classical rough set theory.**

***Index Terms*—Different grades; Graded rough set; multi-granulation rough set; Ordered information system**

I. INTRODUCTION

In 1980s, the Pawlak first proposed the Rough set theory (RST) in [1]. It is a mathematical tool to deal with uncertainty in an information system. Compared with the probability theory, fuzzy set and evidence theory, it have their own advantages in the fields of medical diagnosis, knowledge discovery, image processing and so on.

This theory is built on the basis of the classification mechanism, it's classified as the equivalence relation in a specific universe, and the relation constitutes a partition of the discuss universe. The main idea of RST is the utilize a known knowledge in knowledge base to approximate the inaccurate and indeterminate knowledge. But, there is a severe limitation for Pawlak rough set. That there is no fault tolerance mechanisms between equivalence class and the basic set. Thus, people take considered the degree of overlap of the equivalence class and the basic set in the view of quantitative information. They thought the RST should be improved and expanded that the quantification of particular value are considered. So, Yao et al. investigated the relationship between rough set model and modal logics, the graded rough set(GRS) was proposed by utilizing the modal logics [2]. The absolute quantitative information about knowledge and concepts are described in this model, and expands the Pawlak model. Zhang have accomplished a lot of research studies on graded rough set and related works [3]. Measures $|[x]_R| - |X \cap [x]_R|$ and $|X \cap [x]_R|$ reflect the absolute number of $|[x]_R|$ elements outside and inside X, and called external grade and internal grade, respectively. So, based above absolute numbers and the $\overline{R_k}(X)$ means union of the elements which whose classes's internal grade about$X$ is greater than k; $\underline{R_k}$ means union of the elements which whose classes's external grade about X is at most k [4]. This nature number k is called the grade of GRS. Because we describe the lower and upper approximations in absolute quantitative information so the $\underline{R_k}(X) \subseteq \overline{R_k}(X)$ does not hold, in general. The GRS model which based on two discuss universes was proposed by Liu [5].

Furthermore, the attributes with preference-ordered domains (sometimes it's named criteria) are not studied in the original approaches. In a lot of real applications, we have to faced the issues that one attribute play a crucial role in make decision. For this reason, Yao considered this kind problem that through looking foe the relationship between orderings of attribute values and the objects to mining ordering rules in [6]. For looking for the rules, the general information table be generalized to an information system with order that is ordered information system(OIS). In [7], through taking into account the ordering properties of criteria the RST was expanded by Greco et al.. The expanded model also named dominance-based rough set model (DRSA). Yang investigated the RST approach and reductions based dominance relation in incomplete ordered information system [8]. Xu have systematic studied the rough set in OIS [9]. The main innovation of these works is use a dominance relation replace the indiscernibility relation.

As a useful tool for information processing, based on Zadeh's 'information granularity' [10] the Granular Computing(GrC) was proposed. The GrC is a term of methodologies, tools and techniques for making utilize of granules in the process of solving real problems. In recently decades, more and more scientists focus on the study theory and applications of GrC. In many fields it has been successfully applied such as knowledge discovery, concept formation and machining learning. Based this view, the classical single-granulation Pawlaks rough set have been extended to a multi-granulation rough set model by Qian et al. [11],[12]. And later, many researchers have extended the multi-granulation rough set. Xu et and Wang further investigated a fuzzy multi-granulation RST model in [13], a generalized multi-granulation RST approach [14] and a multi-granulation RST model in ordered information systems. The hierarchical structure properties of the multi-granulation RST and the multi-granulation RST in incomplete information system were investigated by Yang et al. in [15] and [16], respectively. Lin et al. presented a neighborhood-based multi-granulation RST [17]. Furthermore, the properties of multi-granulation RST and the topological structures were deep analyzed by She et al. in [18]. Recently, follow the Xu's work Li et al. developed a further study of multi-granulation T-fuzzy rough set, relationships between multi-granulation and classical T-fuzzy rough set were studied carefully [19].

The rest of the paper is organized by the following way. Some necessary essential concepts are introduced in part 2. In section 3, three types multi-granulation with different grades rough set models are constructed in OIS and their properties are discussed. In section 4, a case about students achievement are studied. The section 5 are the conclusions and the further studies of this topic.

II. Preliminaries

In this section, a few basic notions about RST in OIS [7],[9], graded rough set [2],[3],[4] and multi-granulation rough set [11],[12] are simply reviewed.

A. The rough set model in OIS

An order triple $I^{\succcurlyeq} = (U, AT, F)$ is an ordered information system, let for a criterion $a_l \in AT$ and it's domain is complete pre-ordered through an outranking relation $\succcurlyeq_{a_l}$, then $x \succcurlyeq_{a_l} y$ implies that x is at least no less than y under the criterion a_l. In other words, we can call that x dominates y. One can define $x \succcurlyeq_{a_l} y$ by $f_l(x) \geq f_l(y)$ use the increasing preference, that is $a_l \in AT$, x and $y \in U$. Give a subset A means $A \subseteq AT$ have for all $a_l \in A$, $x \succcurlyeq y$ is equivalent to $x \succcurlyeq_{a_l} y$. There is a another mean that x dominates y under the all rules in A. In summary, an ordered information system can be denoted as $I^{\succcurlyeq} = (U, AT, F)$.

Suppose there is an ordered information system that $I^{\succcurlyeq} = (U, AT, F)$, given $A \subseteq AT$, the $R_A^{\succcurlyeq}$ is a dominance relation in $I^{\succcurlyeq}$ and the relation is $R_A^{\succcurlyeq} = \{(x, y) : f_l(x) \geq f_l(y), \forall a_l \in A, (x, y) \in U \times U\}$, and $U/R_A^{\succcurlyeq} = \{[x]_{R_A^{\succcurlyeq}} : x \in U\}$ is the set of dominance classes induced by a dominance relation $R_A^{\succcurlyeq}$, where $[x]_{R_A^{\succcurlyeq}}$ is called dominance class containing x, and $[x]_{R_A^{\succcurlyeq}} = \{z \in U : (z, x) \in R_A^{\succcurlyeq}\}$. For all $X \subseteq U$, $A \subseteq AT$, the upper and lower approximations are

$$\overline{R}_A^{\succcurlyeq}(X) = \{x \in U : [x]_{R_A^{\succcurlyeq}} \cap X \neq \emptyset\},$$
$$\underline{R}_A^{\succcurlyeq}(X) = \{x \in U : [x]_{R_A^{\succcurlyeq}} \subseteq X\}.$$

When $\overline{R}_A^{\succcurlyeq}(X) \neq \underline{R}_A^{\succcurlyeq}(X)$, in this ordered information system one may call X is a rough set.

B. Graded rough set model

The absolute quantitative information on basic concepts and knowledge granules are the main investigated topics of the graded rough set model. It is also a generalization of the classical rough set. If give a non-negative integer k and it is named 'graded'.

Given an information system $I = (U, AT, F)$, for any $A \subseteq AT, X \subseteq U, k \in \mathbf{N}$ and R_A is a equivalence relation in I. We can get the definition of the graded rough set is

$$\underline{R}_k(X) = \{x \in U : \ |[x]_R| - |[x]_R \cap X| \leq k\},$$
$$\overline{R}_k(X) = \{x \in U : \ |[x]_R \cap X| > k\}.$$

The $\underline{R}_k X$ is the union of the objects that whose equivalence class include the numbers of elements outside X are not more than k. So, the $\overline{R}_k X$ means the union of the objects that whose equivalence class include the numbers of elements inside X are at least k. If $k = 0$, so $\overline{R}(X) = \overline{R}_k(X)$ and $\underline{R}(X)\underline{R}_k(X)$. Thus, the Pawlak rough set is one exceptional situation of this graded rough set model. According to the definition, we can get that the inclusion between $\underline{R}_k(X)$ and $\overline{R}_k(X)$ is not hold in most cases. Then, the upper and lower boundary regions are naturally proposed. And they are represented as $pos(X) = \overline{R}_k(X) \cap \underline{R}_k(X)$, $neg(X) = \sim (\overline{R}_k(X) \cup \underline{R}_k(X))$, the upper boundary region is $Ubn(X) = \overline{R}_k(X) - \underline{R}_k(X)$, the lower boundary region is $Lbn(X) = \underline{R}_k(X) - \overline{R}_k(X)$, and the total boundary region is $bn(X) = Ubn_k(X) \cup Lbn_k(X)$, respectively.

C. Multi-granulation rough set model

We just introduce the models of multi-granulation rough set and the details can be found in references [7].

If there is an information system that $I = (U, AT, f)$, and $A_j \subseteq AT, 1 \leq j \leq m$, m is the number of the considered attribute sets and $[x]_{A_j} = \{y | (x, y) \in R_{A_j}\}$, R_{A_i} is an equivalent relation with respect to the attributes set A_j. We can get the definitions of the optimistic upper and lower approximations of the set $X \in U$ under the A_j are

$$\overline{\sum_{j=1}^{m} A_j^o}(X) = \{x : \ X \cap \bigwedge_{j=1}^{m} [x]_{A_j} \neq \emptyset\},$$
$$\underline{\sum_{j=1}^{m} A_j^o}(X) = \{x : \ \bigvee_{j=1}^{m} [x]_{A_j} \subseteq X\}.$$

The pessimistic lower and upper approximations of a set $X \in U$ about the $A_j \subseteq A, 1 \leq j \leq m$ can be similarly defined by following way.

$$\overline{\sum_{j=1}^{m} A_j^p}(X)(X) = \{x : \ \bigvee_{j=1}^{m} [x]_{A_j} \cap X \neq \emptyset\}.$$
$$\underline{\sum_{j=1}^{m} A_j^p}(X) = \{x : \ \bigwedge_{j=1}^{m} [x]_{A_j} \subseteq X\},$$

Moreover, $\underline{\sum_{j=1}^{m} A_j^o}(X) \neq \overline{\sum_{j=1}^{m} A_j^o}(X)$, $(\underline{\sum_{j=1}^{m} A_j^p}(X) \neq \overline{\sum_{j=1}^{m} A_j^p}(X))$, one can say that X is the optimistic(pessimistic) rough set with respect to multiple equivalence relations or multiple granulations. If not the X is the optimistic(pessimistic) definable set about these multiple equivalence relations or multiple granulations. It's similar to the Pawlak rough set, we can obtain other regions according to the upper and lower approximations, respectively.

Based these above definitions of optimistic and pessimistic multi-granulation rough set, the follow properties are established.

- (1) $\overline{\sum_{j=1}^{m} A_j^o}(X) \subseteq \overline{\sum_{j=1}^{m} A_j^p}(X)$;

- (2) $\underline{\sum_{j=1}^{m} A_j^{o}}(X) \supseteq \underline{\sum_{j=1}^{m} A_j^{p}}(X)$;
- (3) $Bn\sum_{j=1}^{m} A_j^{o}(X) \supseteq Bn\sum_{j=1}^{m} A_j^{p}(X)$.

According to these two types of rough set models, we can see that the optimistic boundary region is smaller and the pessimistic boundary region is bigger than the classical rough set model. In some cases, it can deal with uncertain problems easily.

III. Multi-granulation with different grades rough set in OIS

To study the good points of graded rough set with the multi-granulation environment in OIS, three new types multi-granulation with different grades rough set models are proposed, in this section.

A. The optimistic multi-granulation with different grades rough set in OIS

We combine optimistic multi-granulation and graded rough set model which in different granulation have different graded in OIS.

Definition 3.1.1. There is an ordered information system $I^{\succcurlyeq} = (U, AT, f)$, $A_1, A_2, ..., A_m$ are subset of AT, for any $X \subseteq U$, $k_j \in \mathbf{N}$. The optimistic multi-granulation upper and lower approximation of X with different grades are defined in following way:

$$\underline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(X) = \{x \in U : \bigvee_{j=1}^{m}(|[x]_{A_j}^{\succcurlyeq}| - |X \cap [x]_{A_j}^{\succcurlyeq}|) \leq k_j\},$$

$$\overline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(X) = \sim \underline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(\sim X).$$

We can also define the optimistic multi-granulation with different grades negative region, positive region, and boundary region of X in the ordered information system.

(1) $Pos(X)^{O}_{\sum_{i=1}^{m} k_i} = \underline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X) \cap \overline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X)$;

(2) $Neg(X)^{O}_{\sum_{i=1}^{m} k_i} = \sim (\underline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X) \cup \overline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X))$;

(3) $Lbn(X)^{O}_{\sum_{i=1}^{m} k_i} = \underline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X) - \overline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X)$;

(4) $Ubn(X)^{O}_{\sum_{i=1}^{m} k_i} = \overline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X) - \underline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X)$;

(5) $Bn(X)^{O}_{\sum_{i=1}^{m} k_i} = \overline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X) \bigtriangleup \underline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X)$.

In the graded rough set model the lower approximation is not totally included in upper approximation then, $\underline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X) \subseteq \overline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_i}^{O}(X)$ not holds all the time. So, we describe the boundary region through defined the lower and upper boundary region. The " $\bigtriangleup$ " is the symmetric difference operator of sets. So, the boundary region of X can de described as $Bn(X)^{O}_{\sum_{i=1}^{m} k_i} = Lbn(X)^{O}_{\sum_{i=1}^{m} k_i} \cup Ubn(X)^{O}_{\sum_{i=1}^{m} k_i}$, too.

Theorem 3.1.1 There is an ordered information system $I^{\succcurlyeq} = (U, AT, f)$, $A_1, A_2, ..., A_m$ are subset of AT, for any $X \subseteq U$, $k_j \in \mathbf{N}$. The following properties can be get:

$$\overline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(X) = \{x \in U : \bigwedge_{j=1}^{m}(|[x]_{A_j}^{\succcurlyeq} \cap X|) > k_j\}.$$

Proof. According the Definition 3.1.1 we can get
$\sim \underline{\sum_{j=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_j}^{O}(\sim X) = \sim \{x \in U : |[x]_{A_1}^{\succcurlyeq}| - |[x]_{A_1}^{\succcurlyeq} \cap (\sim X)| \leq k_1 \vee |[x]_{A_2}^{\succcurlyeq}| - |[x]_{A_2}^{\succcurlyeq} \cap (\sim X)| \leq k_2 \vee \cdots \vee \leq k_m\}$
$= \{x \in U : |[x]_{A_1}^{\succcurlyeq}| - |[x]_{A_1}^{\succcurlyeq} \cap (\sim X)| > k_1 \wedge |[x]_{A_2}^{\succcurlyeq}| - |[x]_{A_2}^{\succcurlyeq} \cap (\sim X)| > k_2 \wedge \cdots \wedge |[x]_{A_m}^{\succcurlyeq}| - |[x]_{A_m}^{\succcurlyeq} \cap (\sim X)| > k_m\}$
$= \{|[x]_{A_j}^{\succcurlyeq}| - |[x]_{A_1}^{\succcurlyeq} \cap \sim X| > k_j, \forall j = 1, 2, 3, \cdots, m\}$
$= \{|[x]_{A_j}^{\succcurlyeq} \cap X| > k_j, \forall j = 1, 2, 3, \cdots, m\}$
$= \{x \in U : \bigwedge_{j=1}^{m}(|[x]_{A_j}^{\succcurlyeq} \cap X|) > k_j\}$.

Thus, the theorem is proved. □

Theorem 3.1.2 For an an ordered information system $I^{\succcurlyeq} = (U, AT, f)$, $A_1, A_2, ..., A_m \subseteq AT$, for any $X, Y \subseteq U$, $k_j \in \mathbf{N}$ we have the follow properties:

$$(1)\ \underline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(U) = U; \qquad (2)\ \overline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(\emptyset) = \emptyset;$$

$$(3)\ \underline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(Y) \supseteq \underline{\sum_{i=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_j}^{O}(X), \text{ where } X \subseteq Y;$$

$$(4)\ \overline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(Y) \supseteq \overline{\sum_{j=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_j}^{O}(X), \text{ where } X \subseteq Y;$$

$$(5)\ \underline{\sum_{j=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_j}^{O}(X) \cap \underline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(Y) = \underline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(X \cap Y);$$

$$(6)\ \overline{\sum_{j=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_j}^{O}(X) \cap \overline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(Y) \supseteq \overline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(X \cap Y);$$

$$(7)\ \underline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(X) \cup \underline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(Y) \subseteq \underline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(X \cup Y);$$

$$(8)\ \overline{\sum_{j=1}^{m} R_{A_i}^{\succcurlyeq}}_{k_j}^{O}(X) \cup \overline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(Y) = \overline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_i}^{O}(X \cup Y).$$

Proof: According to the definition, the properties (1) and (2) can be easily proved.

(3) If $X \subseteq Y$ that mens $([x^{\succcurlyeq}]_{A_j} \cap X) \subseteq ([x]_{A_j}^{\succcurlyeq} \cap Y)$ for any $j \in \{1, 2, \cdots, m$. So, $(|[x]_{A_j}^{\succcurlyeq}| - [x]_{A_j}^{\succcurlyeq} \cap X|) \geq (|[x]_{A_j}^{\succcurlyeq}| - [x]_{A_j}^{\succcurlyeq} \cap Y|)$. For any $x \in \underline{\sum_{j=1}^{m} R_{A_j}^{\succcurlyeq}}_{k_j}^{O}(X)$ exist j s.t. $|[x]_{A_j}^{\succcurlyeq}| \leq$

$k_j + |[x]^{\succeq}_{A_j} \cap X|$ so, $|[x]^{\succeq}_{A_j}| - |[x]_{A_j} \cap Y| \le k_j$. That is, $x \in \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(Y)$, namely, $\underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(X) \subseteq \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(Y)$.

(4)For any $x \in \overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(X)$, then, for all j have $|[x]^{\succeq}_{A_j} \cap X| > k_j$. So, $|[x]^{\succeq}_{A_j} \cap Y| > k_j$, hence, $x \in \overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(Y)$. Thus, $\overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(X) \subseteq \overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(Y)$.

(5) For any $X, Y \subseteq U, X \cap Y \subseteq X, Y$, then, according property (3) the " $\subseteq$ " can be proved. For $\forall x \in \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(X) \cap \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(Y)$. Based above definition, we can get at last have one j s.t. $|[x]^{\succeq}_{A_j}| - |[x]^{\succeq}_{A_j} \cap X| \leqslant k_j$ and $|[x]^{\succeq}_{A_j}| - |[x]^{\succeq}_{A_j} \cap Y| \leqslant k_j$. So, it must have one j, s.t. $|[x]^{\succeq}_{A_j}| - |[x]^{\succeq}_{A_j} \cap X \cap Y| \leqslant k_j$. Thus, $x \in \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(X \cap Y)$, means " $\supseteq$ " is hold. Consequently, the prove is fulfilled.
The remaining (6), (7) and (8) can be similarly to prove. □

Theorem 3.1.3 For an an ordered information system $I^{\succeq} = (U, AT, f)$, $A_1, A_2, ..., A_m \subseteq AT$, for any $X, Y \subseteq U$,$k_j \in \mathbf{N}$ we have:

$$\underline{\sum_{i=1}^{m} A^{o}_{j}(X)} \subseteq \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(X),$$

$$\overline{\sum_{i=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(X) \subseteq \overline{\sum_{j=1}^{m} A^{o}_{j}(X)}.$$

Especially, for $\forall k_j = 0, j \in \{1, 2, \cdots, m\}$, then $\underline{\sum_{j=1}^{m} R^{\succeq}_{A_i}}^{O}_{k_j}(X) = \underline{\sum_{j=1}^{m} A^{o}_{j_k}(X)}$, $\overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{O}_{k_j}(X) = \overline{\sum_{j=1}^{m} A^{o}_{j}(X)}$.

So, the multi-granulation with different grades rough set model is also an expansion of multi-granulation RST model.

Proof: According the definition it is easy to prove. □

B. The pessimistic multi-granulation with different grades rough set in OIS

Similarly ways to optimistic multi-granulation rough set model, we discuss pessimistic multi-granulation with different grades RST in OIS.

Definition 3.2.1. There is an ordered information system $I^{\succeq} = (U, AT, f)$, $A_1, A_2, ..., A_m$ are subset of AT, for any $X \subseteq U$, $k_j \in \mathbf{N}$. The pessimistic multi-granulation upper and lower approximation of X with different grades are defined in following way:

$$\underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X) = \{x \in U : \bigwedge_{j=1}^{m}(|[x]^{\succeq}_{A_j}| - |[x]^{\succeq}_{A_j} \cap X|) \le k_j\},$$

$$\overline{\sum_{j=1}^{m} R^{\succeq}_{A_i}}^{P}_{k_j}(X) = \sim \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(\sim X).$$

Moreover, $\underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X) \neq \overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X)$, we say that X is a pessimistic rough set about these multiple grades. If not, one can say that X is pessimistic definable set respect to multiple graded and multiple granulations in ordered information system. Based on the above approximation operators, the other rough regions can be defined like the optimistic case.

Theorem 3.2.1 If $I^{\succeq} = (U, AT, f)$ is an ordered information system, $A_1, A_2, ..., A_m \subseteq AT$, for any $X \subseteq U, k_j \in \mathbf{N}$:

$$\overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X) = \{x \in U : \bigvee_{j=1}^{m}(|X \cap [x]^{\succeq}_{A_j}|) > k_j\}.$$

Proof: The process is similarly to Theorem 3.1.1. □

Theorem 3.2.2 For an an ordered information system $I^{\succeq} = (U, AT, f)$, $A_1, A_2, ..., A_m \subseteq AT$, for any $X, Y \subseteq U$,$k_j \in \mathbf{N}$ we have the follow properties:

$$(1) \underline{\sum_{j=1}^{m} R^{\succeq}_{A_i}}^{P}_{k_j}(U) = U, \qquad (2) \overline{\sum_{j=1}^{m} R^{\succeq}_{A_i}}^{P}_{k_j}(\emptyset) = \emptyset;$$

$$(3) \underline{\sum_{i=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(Y) \supseteq \underline{\sum_{j=1}^{m} R^{\succeq}_{A_i}}^{P}_{k_j}(X), \text{ where } X \subseteq Y;$$

$$(4) \overline{\sum_{j=1}^{m} R^{\succeq}_{A_i}}^{P}_{k_j}(Y) \supseteq \overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X), \text{ where } X \subseteq Y;$$

$$(5) \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X) \cap \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(Y) = \underline{\sum_{j=1}^{m} R^{\succeq}_{A_i}}^{P}_{k_j}(X \cap Y);$$

$$(6) \overline{\sum_{j=1}^{m} R^{\succeq}_{A_i}}^{P}_{k_j}(X) \cap \overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(Y) \supseteq \overline{\sum_{j=1}^{m} R^{\succeq}_{A_i}}^{P}_{k_j}(X \cap Y);$$

$$(7) \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X) \cup \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(Y) \subseteq \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X \cup Y);$$

$$(8) \overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X) \cup \overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(Y) = \overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X \cup Y).$$

Proof: It's similarly to Theorem 3.1.2. □

Theorem 3.2.3 For an an ordered information system $I^{\succeq} = (U, AT, f)$, $A_1, A_2, ..., A_m \subseteq AT$, for any $X, Y \subseteq U$,$k_j \in \mathbf{N}$ we have:

$$\underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}(X) \subseteq \underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X),$$

$$\overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X) \subseteq \overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}(X).$$

Especially, for $\forall k_j = 0, j = 1, 2, \cdots, m$, then $\underline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X) = \underline{\sum_{j=1}^{m} A^{P}_{j_k}(X)}$, $\overline{\sum_{j=1}^{m} R^{\succeq}_{A_j}}^{P}_{k_j}(X) = \overline{\sum_{j=1}^{m} A^{P}_{j}(X)}$.

So, the multi-granulation with different grades rough set is an expansion of the multi-granulation rough set.

Proof: According the definition it is easy to prove. □

C. The mean multi-granulation with different grades rough set in OIS

To discuss the mean graded when there many graded in multi-granulation environment, we define a mean multi-granulation with different grades rough set in OIS.

Definition 3.3.1. There is an ordered information system $I^{\succcurlyeq} = (U, AT, f)$, $A_1, A_2, ..., A_m$ are subset of AT, for any $X \subseteq U$, $k_j \in \mathbf{N}$. The mean multi-granulation upper and lower approximation of X with different grades are:

$$\underline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(X) = \{x \in U : \sum_{j=1}^{m}(|[x]^{\succcurlyeq}_{A_j}| - |[x]^{\succcurlyeq}_{A_j} \cap X|) \leq \frac{\sum_{j=1}^{m} k_j}{m}\},$$

$$\overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(X) = \sim \underline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(\sim X).$$

According the definition of the approximations operator, we can define the other regions like before two cases.

Theorem 3.3.1 For an an ordered information system $I^{\succcurlyeq} = (U, AT, f)$, $A_1, A_2, ..., A_m \subseteq AT$, for any $X, Y \subseteq U$,$k_j \in \mathbf{N}$ we have the follow properties:

$$\overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(X) = \{x \in U : \sum_{j=1}^{m}(|[x]^{\succcurlyeq}_{A_j} \cap X|) > \frac{1}{m}\sum_{j=1}^{m} k_j\}.$$

Proof: According the definition it is easy to prove. □

Theorem 3.3.2 For an an ordered information system $I^{\succcurlyeq} = (U, AT, f)$, $A_1, A_2, ..., A_m \subseteq AT$, for any $X, Y \subseteq U$,$k_j \in \mathbf{N}$ we have the follow properties:

$$(1) \underline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_i}}^{M}_{k_j}(U) = U, \qquad (2) \overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_i}}^{M}_{k_j}(\emptyset) = \emptyset;$$

$$(3) \underline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(Y) \supseteq \underline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_i}}^{M}_{k_j}(X), \text{ where } X \subseteq Y;$$

$$(4) \overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(Y) \supseteq \overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_i}}^{M}_{k_j}(X), \text{ where } X \subseteq Y;$$

$$(5) \underline{\sum_{i=1}^{m} R^{\succcurlyeq}_{A_i}}^{M}_{k_j}(X) \cap \underline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(Y) = \underline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_i}}^{M}_{k_j}(X \cap Y);$$

$$(6) \overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(X) \cap \overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(Y) \supseteq \overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_i}}^{M}_{k_j}(X \cap Y);$$

$$(7) \underline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{P}_{k_j}(X) \cup \underline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(Y) \subseteq \underline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_i}}^{M}_{k_j}(X \cup Y);$$

$$(8) \overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(X) \cup \overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(Y) = \overline{\sum_{j=1}^{m} R^{\succcurlyeq}_{A_i}}^{M}_{k_j}(X \cup Y).$$

Proof: It's similarly to Theorem 3.1.2. □

IV. CASE STUDY

In this section, we study on a students' achievements case based on our previous discussion.

Suppose $I^{\succcurlyeq}$ is an ordered information system about students' achievements and the universe $U = \{x_1, x_2, \cdots, x_{10}\}$ stands for ten students, the set of condition attributes $AT = \{mathematics\ , physical\ , chemistry\ english\ , chinese\ \}$ and denote $A_1 = \{mathematics, physical, chemistry\}$, $A_2 = \{chinese, mathematics, english\}$ are two granules. The value of the attribute "3" means "good", "2" means "medium", "1" means "qualified", and "0" means "filed". Data as shown in table 1.

Table 1 A students' achievements table

	CHN.	Math.	ENG.	Phy.	Chem.
x_1	3	2	3	3	3
x_2	3	2	3	2	2
x_3	2	1	1	2	3
x_4	1	2	2	1	2
x_5	2	3	2	3	2
x_6	2	3	2	3	3
x_7	3	2	3	2	2
x_8	1	0	1	2	3
x_9	1	2	2	1	2
x_{10}	2	1	1	2	3

We first calculate the dominance classes for each object with respect to granule A_1 and A_2.

Suppose $X = \{x_1, x_2, \cdots, x_5\}$, and $k_1 = 1$, $k_2 = 2$ based on the above dominance classes by the definitions we can get the follows results.

Case 1. The result for optimistic multi-granulation with different grades rough set are:

$$\overline{\sum_{j=1}^{2} R^{\succcurlyeq}_{A_i}}^{O}_{k_j}(X) = \{x_4, x_9\}.$$

$$\underline{\sum_{j=1}^{2} R^{\succcurlyeq}_{A_i}}^{O}_{k_j}(X) = \{x_1, x_2, x_3, x_5, x_6, x_7, x_9\},$$

Based the upper and lower approximations other rough regions of X can get as follow.

$Pos(X)^{O}_{\sum_{j=1}^{2} k_j} = \{x_9\}$, $Neg(X)^{O}_{\sum_{j=1}^{2} k_j} = \{x_{8, x_{10}}\}$,

$Lbn(X)^{O}_{\sum_{j=1}^{2} k_j} = \{x_4\}$,

$Ubn(X)^{O}_{\sum_{j=1}^{2} k_j} = \{x_1, x_2, x_3, x_5, x_6, x_7\}$,

$Bn(X)^{O}_{\sum_{j=1}^{2} k_j} = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$.

Case 2. The result for pessimistic multi-granulation with different grades rough set are:

$$\underline{\sum_{j=1}^{2} R^{\succcurlyeq}_{A_i}}^{P}_{k_j}(X) = \{x_1, x_2\},$$

$$\overline{\sum_{j=1}^{2} R^{\succcurlyeq}_{A_i}}^{P}_{k_j}(X) = \{x_1, x_2, x_3, x_4, x_8, x_9, x_{10}\}.$$

The other rough regions of X can be got by the lower and upper approximations as.

$Pos(X)^{P}_{\sum_{j=1}^{2} k_j} = \{x_1, x_2\}$, $Lbn(X)^{P}_{\sum_{j=1}^{2} k_j} = \emptyset$,

$Neg(X)^{P}_{\sum_{j=1}^{2} k_j} = \{x_5, x_6, x_7\}$,

$Ubn(X)^{P}_{\sum_{j=1}^{2} k_j} = \{x_3, x_4, x_8, x_9, x_{10}\}$,

$Bn(X)^{P}_{\sum_{j=1}^{2} k_i} = \{x_3, x_4, x_8, x_9, x_{10}\}$.

Case 3. The result for mean multi-granulation with different grades rough set are:

$$\underline{\sum_{j=1}^{2} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(X) = \{x_1, x_2, x_6\},$$

$$\overline{\sum_{j=1}^{2} R^{\succcurlyeq}_{A_j}}^{M}_{k_j}(X) = \{x_2, x_3, x_4, x_8, x_9, x_{10}\}.$$

According the lower and upper approximations, the other rough regions of X can get as follow.

$Pos(X)^{M}_{\sum_{j=1}^{2} k_j} = \{x_2\}$, $Neg(X)^{M}_{\sum_{j=1}^{2} k_j} = \{x_5, x_7\}$,

$Lbn(X)^{M}_{\sum_{j=1}^{2} k_j} = \{x_1, x_6\}$,

$Ubn(X)^{M}_{\sum_{j=1}^{2} k_j} = \{x_3, x_4, x_8, x_9, x_{10}\}$,

$Bn(X)^{M}_{\sum_{j=1}^{2} k_j} = \{x_1, x_3, x_4, x_6, x_8, x_9, x_{10}\}$.

The three types results are not entirely consistent. Consequently, in different application fields can select different model according the different requirements.

V. Conclusion

The multi-granulation rough set and graded rough set are important expansions of classical rough set and have been applied into many fields. In our study, we integrate the good points of graded rough set and multi-granulation rough set theory in OIS. The main contribution of this paper is that we constructed three different types of multi-granulation with different grades rough set associated with granular computing, in which the upper and lower approximation operators are got by multiple dominance relations, respectively. We have discussed some properties of these three types RST models. Finally, we make a case study evaluate the performance of the proposed properties. This study extended classical rough set in viewpoint of GrC and meaningful compared with the generalization of RST. In our further work, we will study the measure of the three types model and use these approaches to solve the issues in real-world.

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